

Study Guide

For use with pages 187–191

GOAL Find the least common multiple of two numbers.**VOCABULARY**

A **multiple** of a whole number is the product of the number and any nonzero whole number. A multiple that is shared by two or more numbers is a **common multiple**. The least of the common multiples of two or more numbers is the **least common multiple (LCM)**.

The **least common denominator (LCD)** of two or more fractions is the least common multiple of the denominators.

EXAMPLE 1 Finding the Least Common Multiple

You have two alarm clocks set for the same time. The snooze time on one alarm clock is 10 minutes and the snooze time on the other is 15 minutes. If you continue hitting the snooze button on both alarm clocks after they ring the first time together, in how many minutes will the alarm clocks ring together again?

Method 1: List the multiples of each number. Identify the least number that is on both lists.

Multiples of 10: 10, 20, 30, 40, 50, 60
Multiples of 15: 15, 30, 45, 60, 75 } The LCM of 10 and 15 is 30.

Method 2: Find the common factors of the numbers.

$10 = 2 \cdot \underbrace{5}$
 $15 = 3 \cdot \underbrace{5}$ } The common factor is 5.

Multiply all of the factors, using each common factor only once.

$$\text{LCM} = 2 \cdot 3 \cdot 5 = 30$$

Answer: Your alarm clocks will ring together again in 30 minutes.

EXAMPLE 2 Finding the Least Common Multiple of Monomials

Find the least common multiple of $20xy^2$ and $25xy^3$.

$$20xy^2 = 2 \cdot 2 \cdot \underbrace{5} \cdot \underbrace{x} \cdot \underbrace{y \cdot y}$$

$$25xy^3 = \underbrace{5 \cdot 5} \cdot \underbrace{x} \cdot \underbrace{y \cdot y \cdot y}$$

Common Factors are circled and used only once in the LCM.

$$\text{LCM} = 5 \cdot x \cdot y \cdot y \cdot 2 \cdot 2 \cdot 5 \cdot y = 100xy^3$$

Answer: The least common multiple of $20xy^2$ and $25xy^3$ is $100xy^3$.

Exercises for Examples 1 and 2

Find the least common multiple of the numbers or monomials.

1. 5, 17

2. 15, 80

3. 45, 150

4. $24x^{10}y^4$, $42x^7y^6$

5. $13a^3bc$, $26ab^2$

6. $12x^6y^9$, $16yz$

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EXAMPLE 3 Comparing Fractions Using the LCD

Last year, a day spa had 48,000 visitors, including 18,000 men. This year, the day spa had 54,000 visitors, including 24,000 men. In which year was the fraction of men greater?

Solution

(1) Write the fractions and simplify.

$$\text{Last year: } \frac{18,000}{48,000} = \frac{3}{8}$$

$$\text{This year: } \frac{24,000}{54,000} = \frac{4}{9}$$

(2) Find the LCD of $\frac{3}{8}$ and $\frac{4}{9}$.

The LCM of 8 and 9 is 72. So, the LCD of the fractions is 72.

(3) Write equivalent fractions using the LCD.

$$\text{Last year: } \frac{3}{8} = \frac{3 \cdot 9}{8 \cdot 9} = \frac{27}{72}$$

$$\text{This year: } \frac{4}{9} = \frac{4 \cdot 8}{9 \cdot 8} = \frac{32}{72}$$

(4) Compare the numerators: $\frac{27}{72} < \frac{32}{72}$, so $\frac{3}{8} < \frac{4}{9}$.

Answer: The fraction of men was greater this year.

EXAMPLE 4 Ordering Fractions and Mixed Numbers

Order the numbers $2\frac{7}{12}$, $\frac{24}{9}$, and $\frac{13}{6}$ from least to greatest.

(1) Write the mixed number as an improper fraction: $2\frac{7}{12} = \frac{2 \cdot 12 + 7}{12} = \frac{31}{12}$.

(2) Find the LCD of $\frac{31}{12}$, $\frac{24}{9}$, and $\frac{13}{6}$.

The LCM of 12, 9, and 6 is 36. So, the LCD is 36.

(3) Write equivalent fractions using the LCD.

$$\frac{31}{12} = \frac{31 \cdot 3}{12 \cdot 3} = \frac{93}{36} \quad \frac{24}{9} = \frac{24 \cdot 4}{9 \cdot 4} = \frac{96}{36} \quad \frac{13}{6} = \frac{13 \cdot 6}{6 \cdot 6} = \frac{78}{36}$$

(4) Compare the numerators: $\frac{78}{36} < \frac{93}{36}$ and $\frac{93}{36} < \frac{96}{36}$, so $\frac{13}{6} < 2\frac{7}{12}$ and $2\frac{7}{12} < \frac{24}{9}$.

Answer: From least to greatest, the numbers are $\frac{13}{6}$, $2\frac{7}{12}$, and $\frac{24}{9}$.

Exercises for Examples 3 and 4

Order the numbers from least to greatest.

7. $1\frac{3}{16}$, $\frac{59}{44}$, $\frac{71}{60}$

8. $3\frac{4}{17}$, $\frac{68}{21}$, $3\frac{3}{7}$

9. $7\frac{1}{12}$, $\frac{41}{6}$, $7\frac{2}{13}$